2017 Stuvesant Team Contest

The **Stuyvesant Team Contest** is a competition where teams of 3 to 5 work together on a series of 24 questions. The questions are all short response, meaning only the answer is needed. Each question has a point value, ranging from [**5**] (the easiest) to [**12**] (most difficult).

During the contest: You will be given 50 minutes to solve as many problems as your team can. At the beginning, you will start with two problems on separate strips of paper, and every time you want to submit a problem (you may submit problems separately), you can do one of two things:

1. Give in an answer. If you are correct you will receive points equal to the value of the problem. If you are incorrect, you will receive no points

2. Pass the problem (writing "PASS" instead of an answer). You will receive 1 point.

Either way, once you submit the problem, you will get the next problem.

At the end of the contest, you will get also 1 point for each problem you did not submit (including ones you didn't get to), as if you had passed those problems.

No calculators are allowed.

All answers must be in simplest form.

Remember, if you are clueless about solving a problem, you can submit it with no answer for the next problem, getting 1 point.

We hope you enjoy the contest!

1. [**5**] Compute:

$$2 \cdot 4 \cdot 6 - 2 \cdot 4 - 4 \cdot 6 - 6 \cdot 2 + 2 + 4 + 6.$$

Team Name: _____________ Answer: _____________

. . . .

2. [**5**] Compute the least positive integer k such that k is not a multiple of 3 and $10k + 3$ is not prime.

Team Name: _____________ Answer: _____________

. . . .

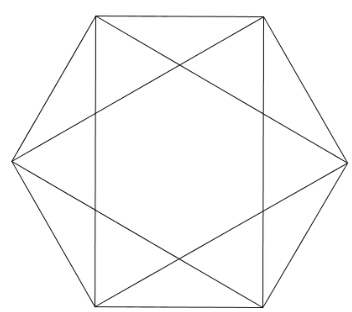
3. [**5**] If $a = 8$, $b = 15$, and $c = 17$, compute:

$$\frac{\frac{a+|a-b|+b}{2} + \left|\frac{a+|a-b|+b}{2} - c\right| + c}{2}$$

Team Name: _____________ Answer: _____________

. . . .

4. [**6**] In the diagram below, the vertices of the smaller regular hexagon are intersections of the diagonals of the larger regular hexagon. Compute the ratio of the area of the smaller regular hexagon to the larger.



Team Name: _____________ Answer: _____________

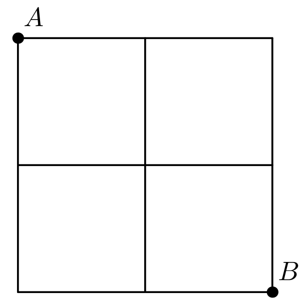
. . . .

5. [**6**] A random number generator outputs the numbers $6, 3$, and 2 with probability $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$, respectively. Compute the average value of the output.

Team Name: _____________ Answer: _____________

. . . .

6. [**6**] Compute the number of paths from A to B along the lattice grid that pass through each of the 9 intersection points at most once.



Team Name: ______________ Answer: ______________

. . . .

7. [**7**] In equilateral triangle ABC with side length 8, D is the foot of the altitude from A to BC and E is the foot of the altitude from D to AC. Compute $[CDE]$.

Note: $[\cdots]$ denotes the area of the polygon in the brackets.

Team Name: ______________ Answer: ______________

. . . .

8. [**7**] Akash, Matthew, and Milan are playing rock-paper-scissors. If each of them randomly chooses their play, compute the probability that Akash beats Matthew, Matthew beats Milan, and Milan beats Akash.

Team Name: ______________ Answer: ______________

. . . .

9. [**7**] Compute

$$(\log_2 2017)(\log_3 2016)(\log_4 2015)\cdots(\log_{2015} 4)(\log_{2016} 3)(\log_{2017} 2).$$

Team Name: ______________ Answer: ______________

. . . .

10. [**8**] In rectangle $ABCD$, E is the center and F is the midpoint of segment AE. If $BF \perp AC$ and $AF = 1$, compute the $[ABCD]$.

Team Name: ______________ Answer: ______________

. . . .

11. [**8**] Compute the maximum number of regions one circle and three lines can divide the plane into. (A region can have finite or infinite area).

Team Name: ______________ Answer: ______________

. . . .

12. [**8**] Let F_n denote the Fibonacci sequence, defined as $F_1 = F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Let s_1 be the sum of the positive integer solutions a to the equation $F_a = a$. Let s_2 be the sum of the positive integer solutions b to the equation $F_b = b^2$. Compute $F_{s_1} + F_{s_2}$.

Team Name: ____________ Answer: ____________

. . . .

13. [**9**] Compute:

$$100^2 - 2 \cdot 99^2 + 3 \cdot 98^2 - 4 \cdot 97^2 + \cdots + 99 \cdot 2^2 - 100 \cdot 1^2.$$

Team Name: ____________ Answer: ____________

. . . .

14. [**9**] Compute the sum of all positive integers a such that $2^a + a^2$ is either a power of 2 or a perfect square.

Team Name: ____________ Answer: ____________

. . . .

15. [**9**] Milan rolls a 20 sided die labeled with the numbers $1, 2, 3, ..., 20$. Matthew rolls a 17 sided die labeled $1, 2, ..., 17$. Compute the probability Matthew's roll is greater than Milan's roll.

Team Name: ____________ Answer: ____________

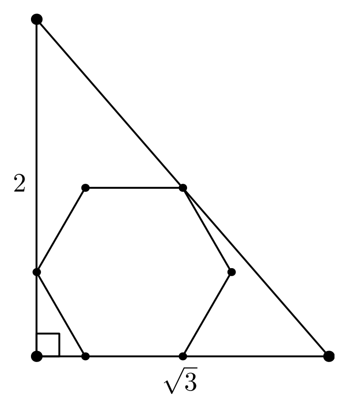
. . . .

16. [**10**] Compute the least positive integer N such that $N^N > 10^{20}$.

Team Name: ____________ Answer: ____________

. . . .

17. [**10**] Compute the side length of the regular hexagon inscribed in a right triangle with legs $\sqrt{3}$ and 2 as shown:



Team Name: ____________ Answer: ____________

. . . .

18. [**10**] Compute the sum of all primes p satisfying:

$$p \mid 3^p + 11^p + 19^p.$$

Note: $a \mid b$ for integers a and b if a is a factor of b.

Team Name: ____________ Answer: ____________

. . . .

19. [**11**] Let $P(x) = x^{2017} - 2017x - 2017$. Let $r_1, r_2, r_3, ..., r_{2017}$ be the zeros of P. If

$$T = r_1^{2017} + r_2^{2017} + r_3^{2017} + \cdots + r_{2017}^{2017},$$

compute the remainder when T is divided by 1000.

Team Name: ____________ Answer: ____________

. . . .

20. [**11**] In $\triangle ABC$, $AC = 20$, $BC = 17$. If $\cos^2 A + \cos^2 B + \sin^2 C = 2$, compute $[ABC]$.

Team Name: ____________ Answer: ____________

. . . .

21. [**11**] Compute the least positive integer n such that:

$$\left\lfloor \frac{n}{4} \right\rfloor + \left\lfloor \frac{n}{4^2} \right\rfloor + \left\lfloor \frac{n}{4^3} \right\rfloor + \left\lfloor \frac{n}{4^4} \right\rfloor + \cdots = 1000.$$

Note: $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

Team Name: ____________ Answer: ____________

. . . .

22. [**12**] In Mr. Cocoros's 5^{th} period calculus class, only 5 students are present. Unfortunately, all 5 have fallen asleep. To wake them up, Mr. Cocoros plans to throw pieces of chalk at them. However, due to his bad aim, Mr. Cocoros misses his target half of the time and hits a random other student. Any student hit by chalk will wake up. If Mr. Cocoros has 5 pieces of chalk to throw and aims them optimally, compute the probability that he will wake up the entire class.

Team Name: ____________ Answer: ____________

. . . .

23. [**12**] In quadrilateral $ABCD$, $AB = CD$ and $\angle BAD + \angle CDA = 90°$. If $BC = 31$ and $[ABCD] = 264$, compute AD^2.

Team Name: ____________ Answer: ____________

. . . .

24. [**12**] An *Additive Magic Square* (AMS) is a 3 by 3 grid filled with distinct positive integers such that the sum of each row, column, and diagonal are the same. A *Multiplicative Magic Square* (MMS) is a 3 by 3 grid filled with distinct positive integers such that the product of each row, column, and diagonal are the same. The minimum possible sum of elements of an AMS is 45. Compute the minimum possible sum of elements of a MMS.

Team Name: ____________ Answer: ____________

. . . .