1. **[10]** A team of 6 distinguishable students competed at a math competition. They each scored an integer amount of points, and the sum of their scores was 170. Their highest score was a 29, and their lowest score was a 27. How many possible ordered 6-tuples of scores could they have scored?

2. **[10]** Let ABC be a triangle with $AB = 13$, $BC = 14$, and $CA = 15$. Let P be a point inside triangle ABC, and let ray AP meet segment BC at Q. Suppose the area of triangle ABP is three times the area of triangle CPQ, and the area of triangle ACP is three times the area of triangle BPQ. Compute the length of BQ.

3. **[10]** I am thinking of a geometric sequence with 9600 terms, a_1, a_2,. . ., a_{9600}. The sum of the terms with indices divisible by three (i.e. $a_3 + a_6 + \cdots + a_{9600}$) is $\frac{1}{56}$ times the sum of the other terms (i.e. $a_1 + a_2 + a_4 + a_5 + \cdots + a_{9598} + a_{9599}$). Given that the terms with even indices sum to 10, what is the smallest possible sum of the whole sequence?

4. **[15]** Let $ABCD$ be a regular tetrahedron with side length $6\sqrt{2}$. There is a sphere centered at each of the four vertices, with the radii of the four spheres forming a geometric series with common ratio 2 when arranged in increasing order. If the volume inside the tetrahedron but outside the second largest sphere is 71, what is the volume inside the tetrahedron but outside all four of the spheres?

5. **[20]** Find the largest number of consecutive positive integers, each of which has exactly 4 positive divisors.

6. **[25]** Let the (not necessarily distinct) roots of the equation $x^{12} - 3x^4 + 2 = 0$ be $a_1, a_2, \ldots, a_{12}$. Compute
$$\sum_{i=1}^{12} |\mathrm{Re}(a_i)|.$$

7. **[25]** Jeffrey is doing a three-step card trick with a row of seven cards labeled A through G. Before he starts his trick, he picks a random permutation of the cards. During each step of his trick, he rearranges the cards in the order of that permutation. For example, for the permutation $(1, 3, 5, 2, 4, 7, 6)$, the first card from the left remains in position, the second card is moved to the third position, the third card is moved to the fifth position, etc. After Jeffrey completes all three steps, what is the probability that the "A" card will be in the same position as where it started?

8. **[30]** In convex equilateral hexagon $ABCDEF$, $AC = 13$, $CE = 14$, and $EA = 15$. It is given that the area of $ABCDEF$ is twice the area of triangle ACE. Compute AB.

9. **[35]** Find all ordered pairs (x, y) of numbers satisfying
$$(1 + x^2)(1 + y^2) = 170$$
$$(1 + x)(-1 + y) = 10.$$

10. **[40]** Call a positive integer "pretty good" if it is divisible by the product of its digits. Call a positive integer n "clever" if n, $n+1$, and $n+2$ are all pretty good. Find the number of clever positive integers less than 10^{2018}. Note: the only number divisible by 0 is 0.

11. **[40]** What is the area in the xy-plane bounded by $x^2 + \frac{y^2}{3} \le 1$ and $\frac{x^2}{3} + y^2 \le 1$?

12. **[45]** Let S be the set of ordered triples $(a, b, c) \in \{-1, 0, 1\}^3 \setminus \{(0, 0, 0)\}$. Let n be the smallest positive integer such that there exists a polynomial, with integer coefficients, of the form
$$\sum_{\substack{i+j+k=n \\ i,j,k\ge 0}} a_{(i,j,k)} x^i y^j z^k$$
such that the absolute value of all the coefficients are less than 2, and the polynomial equals 1 for all $(x, y, z) \in S$. Compute the number of such polynomials for that value of n.

13. **[45]** Let $N \geq 2017$ be an odd positive integer. Two players, A and B, play a game on an $N \times N$ board, taking turns placing numbers from the set $\{1, 2, \ldots, N^2\}$ into cells, so that each number appears in exactly one cell, and each cell contains exactly one number. Let the largest row sum be M, and the smallest row sum be m. A goes first, and seeks to maximize $\frac{M}{m}$, while B goes second and wishes to minimize $\frac{M}{m}$. There exists real numbers a and $0 < x < y$ such that for all odd $N \geq 2017$, if A and B play optimally,
$$x \cdot N^a \leq \frac{M}{m} - 1 \leq y \cdot N^a.$$
Find a.

14. **[50]** Yunseo has a supercomputer, equipped with a function F that takes in a polynomial $P(x)$ with integer coefficients, computes the polynomial $Q(x) = (P(x)-1)(P(x)-2)(P(x)-3)(P(x)-4)(P(x)-5)$, and outputs $Q(x)$. Thus, for example, if $P(x) = x+3$, then $F(P(x)) = (x+2)(x+1)(x)(x-1)(x-2) = x^5 - 5x^3 + 4x$. Yunseo, being clumsy, plugs in $P(x) = x$ and uses the function 2017 times, each time using the output as the new input, thus, in effect, calculating
$$\underbrace{F(F(F(\ldots F(F(}_{2017}x))\ldots))).$$
She gets a polynomial of degree 5^{2017}. Compute the number of coefficients in the polynomial that are divisible by 5.